EQUALITAS
Economics of Inequality and Poverty Analysis

EQUALITAS Working Paper No. 35

Multidimensional Poverty Indices and First Order Dominance Techniques: An Empirical Comparison of Different Approaches

Iñaki Permanyer

M. Azhar Hussain

2015

Multidimensional Poverty Indices and First Order Dominance Techniques: An Empirical Comparison of Different Approaches

Iñaki Permanyer* and M. Azhar Hussain†

Abstract

In this paper we contrast different perspectives to the measurement of multidimensional poverty. Using data from 38 Demographic and Health Surveys around the developing world, we have compared the performance of two broad approaches: multidimensional poverty indices and first order dominance techniques (FOD). Our empirical findings suggest that the FOD approach might be a reasonable cost-effective alternative to the United Nations Development Program (UNDP)'s flagship poverty indicator: the Multidimensional Poverty Index (MPI). To the extent that the FOD approach is able to uncover the socio-economic gradient that exists between countries, it can be proposed as a viable alternative to the MPI with the advantage of not having to rely on the somewhat arbitrary and normatively binding assumptions that underpin the construction of UNDP's index.

Keywords: Multidimensional Poverty Measurement, Poverty Index, First Order Dominance, Cardinal, Ordinal.

JEL codes: I3, I32, D63, O1

* Corresponding Author. Centre d'Estudis Demogràfics, Edifici E-2, Campus de la Universitat Autònoma de Barcelona. E-mail: inaki.permanyer@uab.es

† Department of Society and Globalisation, Roskilde University. E-mail: azharh@ruc.dk

1. Introduction

Poverty eradication continues to be one of the greatest challenges faced by policy-makers around the world. For a long time, poverty has been studied and analyzed on the basis of income distributions (*e.g.*: Sen 1976). Nevertheless, in the last years it is becoming widely acknowledged that both monetary *and* non-monetary attributes are essential to conceptualize and measure individuals' welfare levels (see, for instance, Bourguignon and Chakravarty 2003:26). The measurement of multidimensional poverty is a burgeoning field of research that, despite important advances observed during recent years, is still in its infancy. In this respect, renowned scholars like Erik Thorbecke (2007) have argued that "Most of the remaining unresolved issues in welfare analysis are related directly or indirectly to the multidimensional nature and dynamics of welfare". This paper aims to compare empirically different approaches to the measurement of multidimensional poverty.

Research on the conceptualization and measurement of multidimensional poverty is particularly pertinent at this moment given the fact that international institutions like the European Commission and the United Nations are implementing the multidimensional approach to complement official unidimensional income or consumption poverty measures. Following the definition adopted by the Europe 2020 strategy, Eurostat publishes since 2009 the values of the multidimensional AROPE index (people at-risk-of-poverty rate or social exclusion), and since 2010 the United Nations' Human Development Report (HDR) publishes the values of the so-called 'Multidimensional Poverty Index' (henceforth MPI) for over a hundred countries all over the world (see Alkire and Santos 2010). With the target date of the Millennium Development Goals (MDGs) rapidly approaching, many scholars and policy-makers are engaging in an intense debate on what kind of poverty headline indicator should be the most appropriate to guide poverty eradication strategies in the post-2015 global development agenda. In this context, this paper aims to throw some light on the advantages and disadvantages of currently existing approaches to multidimensional poverty measurement with the aim of informing and enriching these discussions and debates. More specifically, we are interested in critically comparing the results ensuing from two broad perspectives in the measurement of multidimensional poverty: the use of multidimensional poverty indices on the one hand – which are by far the most widely used perspectives implemented in practice so far – and the first order dominance (henceforth FOD) approach on the other hand – a relatively novel and seemingly cost-effective methodology that has the advantage of not relying on the host of debatable assumptions upon which the poverty indices are commonly based.

Multidimensional poverty indices attempt to generalize well-known income poverty measures to a multiple attribute framework by taking into consideration the joint distribution of several variables. For instance, we might be interested not only in the distribution of individuals' income but also in the distribution of those individuals' education or health. Well-known examples of those measures have been proposed by Tsui (2002), Bourguignon and Chakravarty (2003), Chakravarty, Deutsch and Silber

(2008) and, more recently, by Alkire and Foster (2011). The proposal by Alkire and Foster is perhaps the most popular one since the United Nations Development Program (UNDP) adopted it as the MPI in 2010 as a replacement for the Human Poverty Index (HPI). While cardinal measures of this kind can potentially give very precise assessments of existing poverty levels (one might even say overprecise), their construction is based on a wide range of debatable assumptions. To illustrate: important decisions have to be made regarding the choice of the functional form of the index, the weights that are applied to each dimension, the ways in which the different indicators are chosen and normalized or the extent to which deprivations are going to be traded-off between dimensions (i.e.: complementarity and substitutability issues across dimensions), each of which having crucial ethical implications.

When the available variables cannot be measured in a cardinal scale it is common to resort to their ordinal counterparts. The ordinal approach attempts to measure multidimensional poverty when the underlying variables can be completely ordered across different outcomes (i.e.: from 'bad' to 'good'). While ordinal variables might be less precise than their cardinal counterparts when determining the extent of deprivation suffered by individuals (i.e.: it is generally much more difficult to determine whether individuals are 'very' poor or whether slight improvements would lift them out of poverty), they have different advantages. For instance, they are much more reliable and robust and less prone to measurement errors than their cardinal counterparts – to illustrate: the ownership of different assets in the household is much easier to measure than the earnings of its members. In addition, since cardinal poverty measures are ill-equipped to work with ordinal variables – and these variables are often present when assessing multidimensional poverty – it is particularly necessary to define the appropriate ordinal poverty measures. Well-known indices that can be used with ordinal data are the multidimensional headcount ratio H (defined as the proportion of the population who is multi-dimensionally poor) and Alkire and Foster's 'adjusted headcount ratio' M_0 (which corresponds to UNDP's MPI; see Alkire and Foster 2011).

Despite their unquestionable advantages, the definition of existing ordinal multidimensional poverty measures like Alkire and Foster's M_0 still rely on several disputable assumptions that have deep ethical implications. First, one has to decide on how many dimensions an individual has to be deprived in order to be considered as being 'poor' (that is, one has to arbitrarily choose the so-called 'poverty cutoff level'[3]). Second, the deprivations are freely interchangeable as long as they add up to the poverty cutoff level, i.e. if the cutoff is set at two, it is the same to be deprived in dimensions A and B than being deprived in dimensions C and D. While this counting approach is reflective of the current state of the literature, it looks overly simplistic as it just counts the number of deprivations irrespective of their nature. Third, the different dimensions have to be weighted according to the importance that is attached to them. Unfortunately, there are no clear (objective) rules on how to choose the most appropriate poverty cutoff

[3] Technical details on this and other conceptually-related definitions are given in section 2.1.1.

levels and the choice of alternative weighting schemes may alter conclusions with respect to the poverty rankings of the populations we are analyzing[4]. One possible way of overcoming these limitations when using ordinal variables is to make use of the multidimensional first order dominance approach (FOD), which obviates the need for the analyst to apply an arbitrary poverty cutoff level, choosing dimensional weights or imposing a specific social welfare function (see Arndt et al. 2012). As opposed to the previous cardinal and ordinal approaches that generate a poverty index measuring the poverty level of each country, the FOD approach makes all pair-wise comparisons between couples of countries to assess whether one country is at least as poor as another one[5]. The robustness of the FOD approach, however, comes at a price: in some occasions the comparisons between two countries are inconclusive, so the corresponding ranking can be incomplete.

As can be seen, the different methodologies have their advantages and disadvantages. However, since their use has been quite sparse (very often working with a single or a quite reduced number of countries – e.g. Arndt et al. 2012) and disconnected from each other (the papers that use one approach do not use the other[6]), it is entirely unknown whether or not the different approaches provide a coherent and consistent picture of the multidimensional poverty rankings at the international level. The main aim of this paper is to investigate whether the poverty indices and the FOD approaches are essentially conveying the same message or if, on the contrary, they offer complementary views of the prevalence of multidimensional poverty across the developing world. To the extent that current international cooperation, development and aid programs are guided by the rankings derived from these measures, the issues analyzed in this paper are not a mere academic curiosity but have important practical and financial implications for the design of effective poverty eradication strategies. The implications of having one level of association or another between alternative methodologies can be completely different. If the alternative methodologies turn out to be very highly correlated we can safely conclude that our assessments of multidimensional poverty are not highly distorted when using one approach or the other. If this were the case, it would suggest that the information provided by relatively simple ordinal indicators would essentially be the same as the one obtained from the more complex and sophisticated cardinal indicators, so the former would constitute a reasonable, fast, and cost-effective alternative to the

[4]As shown in Cherchye et al. (2008), Permanyer (2011, 2012) and Foster et al (2013), certain composite indices of well-being can be highly sensitive to the choice of alternative weighting schemes.

[5]The fact that the FOD approach is defined for ordinal variables can generate some terminology misunderstandings. In this paper, the 'ordinal approach' only refers to the multidimensional poverty indices that are based on ordinal variables like H or M_0, but *not* to the FOD approach (even if the latter is also based on ordinal information). As shown in detail in section 2, the reason to separate among these perspectives is that the ways in which they approach the measurement problem are fundamentally different.

[6]In a very interesting contribution, Deutsch and Silber (2005) compare the performance of different indices of multidimensional poverty. However, the approach followed in that paper is entirely different from the one taken here. On the one hand, the authors concentrate on a single country (Israel), rather than offering an international perspective. On the other hand, the authors consider poverty indices but not partial order techniques like the ones explored here.

latter. At the other extreme, a lack of significantly positive association between the two approaches would suggest that the cardinal and FOD perspectives might highlight complementary aspects of the same phenomenon: poverty. In addition, such results would raise some red flags that would caution against a thoughtless use of existing multidimensional poverty measures.

The remainder of this paper is organized as follows. In section 2 we present the different methodologies that are being compared and in section 3 we present the data and indicators used in our analysis. Section 4 shows the empirical results and section 5 concludes.

2. Two approaches to the measurement of multidimensional poverty

In this section we present in some detail the definitions that are used in the two approaches to the measurement of multidimensional poverty compared in this paper: the use of indices and the first order dominance perspective.

2.1. Multidimensional poverty indices

If one agrees that poverty is a multidimensional phenomenon, it is quite common to introduce the so-called 'multidimensional poverty indices' in order to measure it. In a nutshell, a multidimensional poverty index summarizes in a single number information concerning individuals, households or other units of analysis across several attributes or dimensions in order to inform the poverty levels in a given population. In formal terms, multidimensional poverty indices are non-trivial functions from a certain multidimensional achievement space (typically a matrix space M with non-negative entries) to the set of real numbers (that is: $P: M \rightarrow R$). Depending on whether all the variables included in M are cardinal or ordinal, we will speak about cardinal or ordinal multidimensional poverty indices respectively.

Following the seminal contribution of Sen (1976), when constructing poverty indices it is almost universal to divide the procedure in two steps: 'identification' and 'aggregation'. In the first step, one must present a criterion to decide who should be considered as being poor. In the second step, once it is decided who is poor and who is not, information regarding the poverty levels of the former is aggregated (i.e.: summarized) into a single number. While the identification step is relatively straightforward in the single dimensional case (one basically sets an income poverty line and checks who is above and who is below the threshold), matters become more complicated in the multidimensional case. In the cases where it is meaningful to aggregate the different attainments into an overall welfare indicator, individuals are identified as 'poor' whenever their aggregate well-being level falls below a given poverty threshold. This approach – which has been advocated by Ravallion (2011) – reduces a multivariate distribution to a single-dimensional distribution and then applies the classical income-poverty tools. Alternatively, whenever aggregation is not meaningful in the attainment space and assuming one is able to define dimension-specific poverty thresholds that allow determining whether individuals are deprived or not in the corresponding dimensions, it is customary to work in the so-called 'deprivation space' – where one takes into account individuals' gaps between observed attainments and the corresponding dimension-specific poverty thresholds. In this setting, one can define the so-called 'union', 'intersection' and 'intermediate' approaches' – which basically identify an individual as being 'poor' depending on the number of dimensions in which she or he is deprived (the so-called 'poverty cutoff level')[7]. So far, the use of the deprivation space as the relevant domain from which to construct multidimensional poverty indices has been predominant in the literature.

Regarding the aggregation step, there is a variety of alternatives that have recently been proposed in the last few years – the interested reader can find an overview of the existing multidimensional poverty indices in Permanyer (2014, Table 1). For the sake of simplicity, in this paper we will only focus on a couple of well-known families of poverty measures: the family of indices suggested by Bourguignon and Chakravarty (2003; denoted as $BC_{\theta,\beta}$) and the class of indices proposed by Alkire and Foster (2011; denoted as M_α). These popular indices have been widely used in empirical applications – the latter has been used by UNDP in the construction of the MPI for the Human Development Report since 2010 – so we will use them as well as representatives of a set of multidimensional poverty indices[8].

2.1.1. The Alkire and Foster method

Let k and n be the number of dimensions and individuals we are taking into account respectively. For each dimension j, denote the corresponding deprivation cutoff (i.e.: the level of achievement considered to be sufficient in order to be non-deprived in that dimension) as z_j (with $z_j>0$). This way, we say that an individual i is 'deprived' in dimension j whenever her/his achievement level x_{ij} (with $x_{ij}\geq 0$) is below z_j and 'non-deprived' otherwise. Having chosen the dimension-specific deprivation cutoffs, one can define the deprivation gap of individual i in dimension j as

$$g_{ij} = max\left\{0, \frac{z_j - x_{ij}}{z_j}\right\}. \qquad [1]$$

Each dimension j is given a weight $w_j > 0$ according to its relative importance. The Alkire and Foster method uses the so-called intermediate approach in the identification of the poor. That is, an individual is considered to be 'poor' if her/his weighted proportion of deprivations is above the poverty cutoff threshold, which is decided by the analyst. In case all dimensions are equally weighted, this criterion means that an

[7] According to the 'union' approach, an individual should be labeled as 'poor' if s/he is deprived in at least one dimension. At the other extreme, the 'intersection' approach states that an individual is 'poor' if s/he is deprived in all dimensions simultaneously. Since these extreme approaches are likely to over-estimate and sub-estimate respectively the set of individuals that should be considered as 'poor' (particularly when the number of dimensions that are being considered is large), Alkire and Foster (2011) proposed a counting approach based on Atkinson (2003) suggesting that an individual is 'poor' when s/he is deprived in an intermediate number of dimensions that has to be decided by the analyst.

[8] The use of other cardinal poverty indices recently proposed in the literature does not add new insights to the main findings of the paper, so we have not presented them for the sake of brevity.

individual is considered to be poor if s/he is deprived in at least c dimensions, where $1 \leq c \leq k$ is an integer chosen by the analyst. If we denote by Q the set of poor individuals, the index proposed by Alkire and Foster (2011) can be written as

$$M_\alpha = \frac{1}{n}\sum_{i \in Q}\sum_{j=1}^{k} w_j g_{ij}^{\alpha} \qquad [2]$$

where $\alpha \geq 0$ is an ethical parameter representing aversion to inequality among the poor. This is a generalization of the well-known income-based FGT poverty measure to the multidimensional case (see Foster, Greer and Thorbecke 1984). For any $\alpha \geq 0$, M_α is bounded between 0 and 1: it takes a value of 0 when no-one is poor and it takes a value of 1 when everyone is poor. When the parameter α equals zero, the measure M_0 can accommodate the ordinal data that commonly arise in multidimensional settings. In that case, g_{ij}^{0} takes a value of 1 whenever individual i is deprived in attribute j and 0 otherwise. Whenever $\alpha > 0$, the index is well-defined for cardinal variables only. While in practice it is quite common to work with the indices M_1 and M_2, it is M_0 which has been used in the construction of UNDP's MPI for its ability to incorporate ordinal variables in its definition.

2.1.2. Bourguignon and Chakravarty

In a pioneering paper, Bourguignon and Chakravarty (2003) discussed some fundamental issues on the measurement of multidimensional poverty and proposed several indices to assess its prevalence among populations. Using the same notation as before, one of the families they suggested can be written as

$$BC_{\theta,\beta} = \frac{1}{n}\sum_{i=1}^{n}\left[\sum_{j=1}^{k} w_j g_{ij}^{\theta}\right]^{\beta/\theta} \qquad [3]$$

where θ and β are strictly positive real numbers. As can be seen, overall poverty is defined as an average of individuals' poverty levels and the latter are defined as a weighted average of their deprivation gaps. Different choices of θ and β lead to alternative averaging functions (e.g.: arithmetic, geometric or hyperbolic means when $\theta=1, 0, -1$ and $\beta=1$ respectively). In all cases, $BC_{\theta,\beta}$ is bounded between 0 and 1 with the same interpretation as before. The identification method implicitly used in $BC_{\theta,\beta}$ is the so-called union approach, that is: if an individual is deprived in any dimension then s/he is considered to be 'poor' (see footnote #7).

Interestingly, while equation [2] is quite flexible for the 'identification step', equation [3] is quite flexible in the 'aggregation step'. In some respects, both measures can be thought as a generalization of the other. When the union approach is used in [2], $BC_{\theta,\beta}$ arises as a specific member of the family M_α as long as $\theta=\beta$. And vice versa: when $\theta=\beta$ in equation [3], M_α arises as a specific member of the family $BC_{\theta,\beta}$ as long as the union approach is used in the identification step.

The construction of M_α and $BC_{\theta,\beta}$ relies on a long list of debatable assumptions with important ethical implications. The normalization procedures that underpin the creation of the deprivation gaps g_{ij} are unlikely to be innocuous to the measurement process, and the same can be said about the choice of weights w_j and the functional form of the indices that aggregate individuals' deprivation gaps. For both families of indices, it is implicitly taken for granted that the degree of complementarity and substitutability among deprivations in different dimensions is exactly the same, a heroic assumption to say the least. As an alternative measurement perspective that circumvents many of the aforementioned problems, different authors have suggested the use of the FOD approach, an issue to which we turn now.

2.2. The FOD approach

In contrast to the above measures, the first order dominance (FOD) approach does not measure poverty cardinally, that is: as opposed to the other poverty indices, the FOD approach has been designed to compare couples of countries bi-laterally rather than assigning a real number for each country indicating the corresponding poverty levels. For the sake of simplicity we start by briefly reviewing the well-known theory of one-dimensional first order dominance. Consider a simple model in which, for each individual, there is only a finite set of ordered outcomes S. In this context, the distribution of well-being of some population is described by a probability mass function f over S (that is: $\sum_s f(s) = 1$ and $f(s) \geq 0$ for all s in S). If there are two such distributions f and g, we say that f first order dominates g if any of the following equivalent conditions hold:

(i) g can be obtained from f after a finite sequence of bilateral transfers of density to less desirable outcomes.

(ii) $\sum_s f(s)h(s) \geq \sum_s g(s)h(s)$ for any non-decreasing real function h.

(iii) $F(s) \leq G(s)$ for all s in S, where F and G are the cumulative distribution functions associated to f and g respectively.

In words, condition (i) says that one distribution FOD another if one could hypothetically move from one distribution to the other by sequentially shifting population mass in the direction from a better outcome to a worse outcome. Therefore, whenever f FOD g, the population represented by f is unambiguously better off than the one represented by g.

The generalization of these results to the multidimensional case is well established (e.g. Lehmann 1955, Strassen 1965). Assume now that f and g are multidimensional probability mass functions over a finite subset S of R^m for some natural number m. In this context, the definition of FOD is written exactly as in the single-dimensional case regarding the first two conditions (i) and (ii), while the third one is rewritten as:

(iii') $\sum_{s \in T} g(s) \geq \sum_{s \in T} f(s)$ for any comprehensive set $T \subseteq S$ (T is comprehensive if $s \in T, t \in S$ and $t \leq s$ implies $t \in T$).

As can be seen, the FOD approach does not rely on weighting schemes or on assumptions regarding substitutability / complementarity relationships between welfare dimensions. However, the FOD approach is not always able to determine a ranking when two given countries are compared because conditions (i'), (ii') and (iii') might fail to be satisfied. When this happens, the FOD criterion remains inconclusive. In addition, the FOD procedure provides no sense as to the degree of dominance (or similarity) between two populations – that is, one does not know if the dominance of population *A* over population *B* would be preserved even after large deteriorations in the individual welfare indicators of population *A*, or whether the conclusion of dominance is so fragile that slight deteriorations in any welfare indicator for population *A* would lead to an inconclusive outcome. In order to mitigate this shortcoming, Arndt et al. (2012) use bootstrap techniques to generate estimates of the probability that one population *A* dominates another population *B*. In this paper, *for each country pair* we have repeated the FOD exercise with 100 bootstrapped samples reflecting the survey sample designs of each survey in each year. Among these 100 samples, we have then calculated in what proportion the FOD relationship actually holds, a number that might be interpreted as the probability that one country dominates the other. If one is willing to accept these probabilities as measures of welfare, it is possible to cardinally rank different populations without imposing any weighing structure on the chosen ordinal indicators.

2.2.1. The binary case

In order to illustrate these definitions we focus on the case of three binary 0–1 dimensions that will be used in the empirical section of the paper. In this case, the space of outcomes has 2^3=8 elements: *S*={(0,0,0), (1,0,0), (0,1,0), (0,0,1), (1,1,0), (1,0,1), (0,1,1), (1,1,1)}, where "1" denotes the "good" outcome (e.g: non-deprived) and "0" the bad one (e.g: deprived). Therefore, (0,0,0) denotes the outcome where someone is deprived in all three dimensions simultaneously, (1,1,0) means that someone is only deprived in the third dimension, and so on. In this framework, condition (iii') can be written as follows. If *f* and *g* are two probability mass functions on *S*, we say that *f* first order dominates *g* if the following eleven inequalities (1)–(11) are jointly satisfied:

(1) $g(0,0,0) \geq f(0,0,0)$

(2) $g(0,0,0) + g(1,0,0) \geq f(0,0,0) + f(1,0,0)$

(3) $g(0,0,0) + g(0,1,0) \geq f(0,0,0) + f(0,1,0)$

(4) $g(0,0,0) + g(0,0,1) \geq f(0,0,0) + f(0,0,1)$

(5) $g(0,0,0) + g(1,0,0) + g(0,1,0) + g(1,1,0) \geq f(0,0,0) + f(1,0,0) + f(0,1,0) + f(1,1,0)$

(6) $g(0,0,0) + g(1,0,0) + g(0,0,1) + g(1,0,1) \geq f(0,0,0) + f(1,0,0) + f(0,0,1) + f(1,0,1)$

(7) $g(0,0,0) + g(0,1,0) + g(0,0,1) + g(0,1,1) \geq f(0,0,0) + f(0,1,0) + f(0,0,1) + f(0,1,1)$

(8) $g(0,0,0) + g(1,0,0) + g(0,1,0) + g(0,0,1) + g(1,1,0) + g(1,0,1) \geq$

$f(0,0,0) + f(1,0,0) + f(0,1,0) + f(0,0,1) + f(1,1,0) + f(1,0,1)$

(9) $g(0,0,0) + g(1,0,0) + g(0,1,0) + g(0,0,1) + g(1,1,0) + g(0,1,1) \geq$

$f(0,0,0) + f(1,0,0) + f(0,1,0) + f(0,0,1) + f(1,1,0) + f(0,1,1)$

(10) $g(0,0,0) + g(1,0,0) + g(0,1,0) + g(0,0,1) + g(1,0,1) + g(0,1,1) \geq$

$f(0,0,0) + f(1,0,0) + f(0,1,0) + f(0,0,1) + f(1,0,1) + f(0,1,1)$

(11) $g(0,0,0) + g(1,0,0) + g(0,1,0) + g(0,0,1) + g(1,1,0) + g(1,0,1) + g(0,1,1) \geq$

$f(0,0,0) + f(1,0,0) + f(0,1,0) + f(0,0,1) + f(1,1,0) + f(1,0,1) + f(0,1,1)$

The FOD concept is illustrated in Figure 1 using three hypothetical welfare indicators denoted as I, II and III. The entries in the cells represent the probabilities for the joint distribution of all three indicators. We consider three hypothetical distributions: *q*, *r* and *s*. We see that *q* does not FOD *r*, and vice versa. This is because although $r(0,0,0) > q(0,0,0)$ giving 30>20 (e.g. condition (1) is fulfilled), we also see that $r(0,0,0) + r(1,0,0) + r(0,1,0) + r(1,1,0) < q(0,0,0) + q(1,0,0) + q(0,1,0) + q(1,1,0)$ giving 34<35, which is a violation of condition (5). The lack of FOD is illustrative of the fact that *q* and *r* alters rank depending on evaluation criteria. For instance *q* is better than *r* if the criterion is minimization of the group with members who are simultaneously worse off in all dimensions ($q(0,0,0) < r(0,0,0)$ giving 20<30). But *r* is better than *q* if the criterion is instead maximization of population shares characterized by good outcomes in the three dimensions separately; shares with good outcomes in dimensions I-III is 66, 67 and 66 % for distribution *r*, compared with 65 % for each dimension in distribution *q*.

Looking next at welfare distributions *r* and *s* we also see that none dominates the other (e.g. lack of FOD). Condition (1) is fulfilled since $r(0,0,0) > s(0,0,0)$ giving 30>10. But the last condition is not fulfilled (37<40). Again the problem that arises is that domination depends on evaluation criteria. Distribution *s* is better than *q* if the criterion is maximization of population shares characterized by good outcomes in the three dimensions separately; shares with good outcomes in dimensions I-III is 75 % in each in distribution *s*, while the population shares are 66, 67 and 66 % for distribution *r*. On the other hand *r* is better than *s* if we want to maximize the group who simultaneously do well on all three criteria, e.g. $r(1,1,1) > s(1,1,1)$ giving 63>60.

The remaining comparison is between distributions *q* and *s*. If we insert Figure 1 probabilities in conditions (1)-(11) we see all are met, and we can therefore conclude that *s* FOD *q*. To reach that conclusion we can also use the intuitive strategy where we move probability mass from better to worse to see if one distribution (the dominated one) can be generated from the other (the dominating one). In this case we just need to move 10 % from the best outcome (1,1,1) to the worst outcome (0,0,0) in distribution *s*, which will result in distribution *q*.

[[[Figure_1, see page 29]]]

2.2.2. Definitions

Let X denote the set of countries we are working with. The FOD criterion generates a binary relation on X[9] that will be denoted by $\leq_{FOD}$. Whenever a country x FOD dominates another country y, we will write $y \leq_{FOD} x$. It is straightforward to check that $\leq_{FOD}$ is *reflexive* ($x \leq_{FOD} x$ for all x in X) and *transitive* (if $x \leq_{FOD} y$ and $y \leq_{FOD} z$ then $x \leq_{FOD} z$), so $\leq_{FOD}$ is a *preorder*. Since the FOD approach might reach an indeterminate outcome for certain couples of countries, we say that the preorder is not necessarily *complete* (i.e.: there may exist some countries x and y for which neither $x \leq_{FOD} y$ nor $y \leq_{FOD} x$). For any $x \in X$, we define the *up-set* of x as the set of elements $y \in X$ such that $x \leq_{FOD} y$: it will be denoted by $x^{\uparrow}$. Analogously, the *down-set* of x is the set of elements $y \in X$ such that $y \leq_{FOD} x$: it is denoted by $x^{\downarrow}$.

The set of couples of countries that are comparable in terms of $\leq_{FOD}$ will be denoted by C_{FOD}, that is:

$$C_{FOD} \coloneqq \{(A, B) \in X \times X | A \leq_{FOD} B \;\; or \;\; B \leq_{FOD} A\}. \qquad [4]$$

The complement of this set – which will be denoted by N_{FOD} – is the set of couples of countries that cannot be ranked by $\leq_{FOD}$. The sets C_{FOD} and N_{FOD} are a partition of $X \times X$, that is: $C_{FOD} \cup N_{FOD} = X \times X$ and $C_{FOD} \cap N_{FOD} = \emptyset$.

Observe that any poverty index P also generates a preorder on X defined as follows:

$$x \leq_P y \;\leftrightarrow P(y) \leq P(x). \qquad [5]$$

That is: $\leq_P$ orders countries with respect to the poverty levels determined by P. Since the poverty levels are real numbers and any couple of real numbers is always comparable, the preorder $\leq_P$ is complete. Therefore $C_P = X \times X$ and $N_P = \emptyset$ for all P. Using the poverty indices defined in section 2.1, in this paper we will also consider the complete preorders $\leq_{M0}$, $\leq_{M1}$, $\leq_{M2}$, $\leq_{BC21}$ and $\leq_{BC12}$.

3. Data and indicators

In order to compare multidimensional poverty measurement approaches across the developing world we have assembled 38 Demographic and Health Surveys (DHS)[10] primarily undertaken between 2004 and 2009 (totaling 312,514 observations among the 38 surveys). The final list of DHS surveys included in this paper has been determined by three simultaneous factors: 1. We have chosen the DHS surveys that were used in the

[9] A binary relation on a set X is a subset of the Cartesian product $X \times X$.

[10] The 38 countries included in the dataset and the year in which the DHS was taken are: Albania (2008), Armenia (2005), Azerbaijan (2006), Bangladesh (2007), Benin (2006), Bolivia (2008), Cambodia (2005), Cameroon (2004), Colombia (2010), Egypt (2008), Ethiopia (2005), Ghana (2008), Guinea (2005), Haiti (2005), Honduras (2005), Jordan (2009), Kenya (2008), Lesotho (2004), Liberia (2007), Madagascar (2008), Malawi (2004), Maldives (2009), Mali (2006), Namibia (2006), Nepal (2006), Nicaragua (2001), Niger (2006), Nigeria (2008), Peru (2004), Rwanda (2005), Sao Tome and Principe (2009), Senegal (2005), Sierra Leone (2008), Swaziland (2006), Timor-Leste (2009), Uganda (2006), Zambia (2007), and Zimbabwe (2005).

construction of the official MPI, an issue that ensures comparability with UNPD's well-known measure; 2. We have not used other data sources like MICS (Multiple Indicator Cluster Survey) or WHS (World Health Surveys) to avoid the comparability problems that might arise if we used alternative data sources; 3. We have only chosen the surveys with the necessary sub-national information (strata and primary sampling units) to construct our bootstrapped FOD estimates. All in all, we have obtained an extensive sample including countries from all major regions of the world.

The DHS are nationally representative surveys with large sample sizes and questionnaires that are virtually identical across time and countries. In most surveys, households are selected based on a standard stratified and clustered design, and, within the household, one woman, aged 15-49, is selected at random as the focus of the interview. In addition, all living children up to a given age (usually 60 months, but sometimes 36 months) born to that woman are weighed and measured. These surveys have been widely used by different researchers to measure poverty levels in developing countries (see, among others, Sahn and Stifel 2000 or Duclos, Sahn and Younger 2006).

This constitutes a formidable database with more than enough variables to compute non-monetary multidimensional poverty measures on the basis of the cardinal and FOD perspectives. In this paper, the poverty measures are calculated at the household level – a choice that, while driven by data constraints, is intuitive and facilitates comparisons with the original MPI of the UNDP. Mimicking the methodology used in the definition of the MPI, here we concentrate on three dimensions: Education, Standard of Living and Health. For the education component we will use the indicator 'Years of Schooling', which acts as a proxy for the level of knowledge and human capital of household members. While this indicator has different shortcomings (e.g.: does not capture quality of education or level of knowledge attained), it is a robust and widely available indicator that provides the closest feasible approximation to levels of education for household members. When measuring households' deprivation in terms of 'Years of Schooling', we focus on the highest value of that variable among the different household members – thus following the approach taken in the construction of the MPI. To determine whether a given household is deprived or not in terms of education it is customary to set the poverty threshold at five years of education (i.e: z_1=5; see, for instance Grimm et al. (2008) or the construction of UNDP's MPI).

Our indices also include a standard of living component. Since the DHS were not designed for economic analysis, there are no data on income or expenditure – the standard money metric measures of standard of living. Despite this drawback, the DHS do contain information on household assets that can be employed to represent an alternative to a monetary metric. In the absence of income or expenditure data, we derive a composite welfare index constructed from the households' asset information available in the DHS. Asset indices have been widely used in the literature (e.g.: Filmer and Pritchett 2001, Sahn and Stifel 2000, 2003, Grimm et al. 2008, Harttgen and Klasen 2011, Permanyer 2013, 2014) and their advantages and disadvantages are well known (Filmer and Scott 2012 provide an excellent survey in this regard). In this paper, the asset index has been constructed with 13 items[11]. In the related literature, it is common to draw the poverty threshold z_2 for the standard of living distribution at the 25th, 33rd or the 40th percentiles (e.g.: Sahn and Stifel 2000, 2003). In this paper we report the values corresponding to the 33rd percentile (the conclusions remain essentially the same for the other cutoffs).

The health status of individuals is a crucial ingredient that should be taken into account when assessing overall well-being or deprivation levels. Unfortunately, different authors acknowledge that health is the most difficult dimension to measure in the assessment of multidimensional poverty because of the lack of appropriate data. Mimicking the MPI methodology, we use information on the nutritional status of individuals to estimate deprivations in the health dimension. The indicator that will be used to assess individuals' malnutrition is the Body Mass Index (BMI), which is defined as the ratio between weight (measured in kilograms) and the square of height (measured in meters)[12]. While the BMI is far from being 'the' perfect health indicator (for instance, it does not reflect micronutrient deficiencies and, since it might be influenced by alimentary disorders, fashion norms or recent illnesses, it is not always related to poverty), there are several reasons why it is meaningful to include it in our assessment of poverty levels: (1) Adults who are malnourished are susceptible of different health disorders, they are less able to concentrate and learn and may not perform as well in work (Alkire and Santos 2010:12); (2) Anthropometric data is particularly interesting for poverty analysis because it also reflects both nutritional and health status (including the availability and quality of health care services) and it accounts for caloric consumption relative to needs (Sahn and Younger 2009); (3) Even in the presence of overweight, BMI can serve as a welfare measure if considered to reflect command over resources that allow the consumption of food and other types of nutritional intake (Araar et al. 2009); (4) Unlike monetary poverty, the BMI is less prone to measurement errors and these are likely to be random (Sahn and Younger 2009); (5) In developing countries it is typically the case that high body mass is associated with more affluent individuals (Sobal and Stunkard 1989, McLaren 2007, Wittenberg 2013). Therefore, many studies conclude that in the context of developing countries, it is natural to interpret the BMI as monotonically increasing with overall well-being (as long as it does not achieve extremely large values associated with unhealthy outcomes, like severe obesity). When measuring households' deprivation in the dimension of health we focus

[11] The list of assets used in this paper is the following: 1. Electricity: The household has electricity; 2. Sanitation (toilet facility): The household sanitation facility is improved and not shared with other households; 3. Water: the household does have access to clean drinking water, or clean water is less than 30 minutes walking from home; 4. Floor: The household has no dirt, sand or dung floor; 5. Roof: The household has finished roofing; 6. Walls: The household has finished walls; 7. Cooking fuel: The household does not cook with dung, wood or charcoal; 8. Radio: The household has a radio; 9. TV: The household has a TV; 10. Telephone: The household has a telephone; 11. Refrigerator: The household has a refrigerator; 12. Bike: The household has a bike; 13. Motor vehicle: The household has a motor vehicle (motorbike, car, truck).

[12] As is known, the MPI also includes information on child nutrition. However, that information is missing in many households and its inclusion renders comparisons between households with and without children more problematic on conceptual grounds. For these reasons, we have preferred not to include that indicator in our assessment of multidimensional poverty.

on the lowest BMI among all household members – a criterion that corresponds to that of the MPI, where all household members are considered to be deprived in nutrition if at least one undernourished person is observed in the household. In the literature, it is common to set the BMI threshold to determine whether individuals are malnourished or not at a value of 18.5 (that is: z_3=18.5).

While information regarding education and wealth was available in virtually every household, the BMI variable had a relatively large amount of missing cases (on average 36% of the households missed that information). In order to test whether this might affect our results we compared the distributions of education and standards of living for the two groups of households: those with BMI information and those without it. Observing Figures 2 and 3, one concludes that households with data on the BMI tend to have a slightly higher standard of living and they tend to be slightly more educated than those households that do not have that information. Since poverty levels are estimated on the basis of households with complete information only (i.e: those households having information on education, standards of living *and* health), our estimates of *absolute* poverty levels might be downwardly biased – an issue that is likely to affect UNDP's MPI as well. Observe, however, that this bias will affect *all* poverty measures considered in this paper *in the same direction*, so it is likely that the *relative* position between countries remains unaffected. Therefore, we do not expect that such slight biases should have an important effect on the relationship between the different approaches to the measurement of poverty – whose study is the main aim of the paper.

[[[Figures_2_and_3, see pages 29-30]]]

We conclude this section observing that the poverty thresholds z_1, z_2 and z_3 are used not only for the construction of the multidimensional poverty indices shown in equations [2] and [3] but also to dichotomize our three well-being dimensions to the {0,1}-scale for the FOD approach (0 indicating the "bad" outcome (i.e.: deprived) and 1 the "good" one (i.e.: non-deprived)).

4. Empirical analysis

In this section we present the empirical findings of the paper using the aforementioned 38 Demographic and Health Surveys. In the first subsection we present the results for the poverty indices, in the second one the results for the FOD approach and in the third one we compare the performance of the different approaches simultaneously.

4.1. Results for multidimensional poverty indices

Table 1 shows the values of the poverty indices M_α and $BC_{\theta,\beta}$ for different specifications of the parameters α, θ and β for the 38 countries included in our study. More specifically, we have chosen the values of α=0, α=1, α=2 and the couples (θ, β) = (1, 2) and (θ, β) = (2, 1). Except for the case α=0 – which corresponds to the ordinal Alkire and Foster's measure M_0 – all other cases correspond to cardinal poverty measures. Along with the values of the different poverty indices, Table 1 also presents in parentheses the corresponding country rankings (with the countries having lower poverty levels being placed at the better – i.e. smallest in number – positions in the ranking). The ranges of observed values for the different measures are quite heterogeneous; for M_0, M_1, M_2, $BC_{2,1}$, $BC_{1,2}$ they are [0.031, 0.665], [0.016, 0.4], [0.011, 0.354], [0.028, 0.526] and [0.005, 0.219] respectively. Interestingly, the minimum value is systematically attained by Albania (except when one uses M_2) and the maximum value is always attained by Niger, the poorest country in our sample according to these measures.

[[[Table_1, see page 25]]]

When exploring the relationship between these different indices, it turns out that all cardinal poverty measures explored in this paper are highly correlated among themselves. As shown in the scatterplot matrix of Figure 4, the cardinal measures M_1, M_2, $BC_{2,1}$ and $BC_{1,2}$ are essentially conveying the same information. Interestingly, the ordinal poverty measure M_0 stands out of the rest. While the association between M_0 and the rest of cardinal measures is clearly positive, the dispersion is markedly higher – therefore indicating that the poverty rankings generated by cardinal and ordinal measures have a non-negligible degree of disagreement.

Another way of approaching the same issue is to investigate the extent to which the different poverty indices considered in this paper rank couples of countries in a consistent way or not. For the case at hand, since we are dealing with 38 countries, there are 38·37/2=703 couples of countries. It turns out that the five poverty indices considered in this paper consistently rank 591 of these couples. That is: in 84.07% of the cases there is a simultaneous agreement among the five measures when determining which of the corresponding two countries has the highest poverty levels – a remarkably high degree of agreement. In this context, it is revealing to examine the corresponding degree of agreement when one of the five poverty indices is dropped at a time from the list. When we look at the agreement reached between M_1, M_2, $BC_{2,1}$ and $BC_{1,2}$ (that is, when M_0 is dropped from the list), we obtain a much higher value of 94.45%. Dropping M_1, M_2, $BC_{2,1}$ and $BC_{1,2}$, the corresponding agreement levels are: 84.07%, 84.78%, 84.07% and 84.21%. Interestingly, it is only when M_0 is dropped from the list that the agreement between the remaining indices increases substantially – a result that confirms the dissimilarity between our ordinal index and the cardinal ones observed in Figure 4.

[[[Figure_4, see page 30]]]

As an external consistency check to validate the reasonableness of the measures introduced in this paper, in Table 1 we have also included the values of the official UNDP's MPI. As shown in Figure 5, both our ordinal measure M_0 and the MPI tend to rank countries quite in the same way: the correlation coefficient between the two measures equals 0.95. While reassuring – this result suggests that the measures proposed in the paper are within the bounds of "reasonableness" – such relationship is not surprising given the way in which M_0 has been defined (see section 3).

[[[Figure_5, see page 31]]]

4.2. Results for the FOD approach

As indicated in section 2.2.1, with three binary indicators of deprivation there are 2^3=8 possible welfare indicator combinations. The share of households falling into each of the 8 categories for each country is shown in Table 2. On average (un-weighted by countries' overall population) nearly 42% of the population experiences the best welfare combination, which means they are not deprived in any of the three dimensions (1,1,1). We see a large variation across countries with Albania and Egypt around 90 %, while Niger, Ethiopia and Rwanda below 10%. On average some 6% of people experience the worst welfare combination, that is: deprivation with respect to all three dimensions simultaneously (0,0,0). Also high variation is seen for this combination since Madagascar, Ethiopia and Niger are all above 20%, while Albania and Azerbaijan have none. The remaining slightly more than half of the population experiences intermediate welfare combinations (a mixture of 0/deprivation and 1/non deprivation), which in many cases cannot be internally ranked without assumptions regarding each dimension's importance. This can be 'solved' through applying a weighting scheme, like in the other (cardinal) poverty measures, but is not done in the FOD approach. The least common combination is deprivation in education and health and non-deprivation in wealth (0,1,0).

[[[Table_2, see page 26]]]

With this information on joint probabilities, we can perform all pair-wise FOD comparisons for the 38 countries included in our sample. With the many conditions to be fulfilled to obtain FOD (the 11 equations of section 2.2.1 have to be satisfied simultaneously), one could perhaps expect that it would rarely appear, but in fact we do see a number of FOD – the results are shown in Table 3. To be exact there are 350 instances of FOD among the total of 703 country pairs, which means 350 couples of countries are ranked out of a maximum of 703 (49.8%). That is: if one picks a couple of countries at random, there is a 0.5 probability that they can be ranked vis-à-vis each other. The lack of FOD for half of the country comparisons should not be seen as a failure of the FOD methodology, but it should rather be seen as a strength since, as illustrated in Figure 1, lack of FOD exactly occurs in situations where rankings are dependent on differing evaluation criteria, which means rankings are not robust or consistent in these cases and the analyst should be careful in making the ranking anyway. Since we are dealing with 38 countries, the maximum number of times a country can dominate another one is 37. As can be seen in Table 3, the highest observed number of dominations is for Albania, which dominates 30 countries, while Egypt is close by with 28 dominations. Six countries do not dominate other countries (Bangladesh, Ethiopia, Madagascar, Niger, Senegal and Uganda). The highest number of times a country is being dominated is 29 (Ethiopia), while Madagascar and Niger are dominated by 26 countries. Six countries are not dominated by other countries (Albania, Armenia, Azerbaijan, Egypt, Jordan and Lesotho). As expected, there is a clear negative correlation (-0.77) between the number of dominations and the number of times being dominated – the more a country dominates, the less it is dominated, and vice versa.

[[[Table_3, see page 27]]]

As indicated before, the FOD procedure generates a binary outcome (dominance vs. non-dominance) but provides no sense as to the *degree* of dominance between two countries. To overcome this limitation, the bootstrap techniques introduced in section 2.2 estimate *for each country pair* the probability that one country dominates the other. Interestingly, when the bootstrapped probabilities are compared with the crisp 0–1 outcomes of the (standard) FOD procedure shown in Table 3, the results are extremely similar (the correlation coefficient equals 0.99) – so it is quite redundant to generate an equivalent table with such probabilities. Instead, in Figure 6 we show two histograms: in the first one we show the distribution of bootstrap probabilities for the pairs of countries without FOD dominance among the actual samples and in the second one we show the same distribution for those pairs of countries where FOD dominance occurs in the actual samples. As indicated above, in our dataset there are 350 country pairs where FOD occurs among the actual samples. In 299 of these cases, FOD dominance was also observed in all 100 bootstrapped replications. In the remaining 51 country pairs, the average proportion of FOD dominance out of 100 replications was 0.815, with the lowest at 0.42 and the highest at 0.99. Among the 353 country pairs where FOD dominance does not occur using the actual samples, it turns out that in 300 cases the bootstrapped probabilities are actually 0 (i.e: in none of the corresponding 100 bootstrapped replications did we observe FOD dominance). In the remaining 53 cases the average proportion of FOD dominance out of 100 replications was 0.182, with the lowest at 0.01 and the highest at 0.53. These results suggest that the conclusions reached by the standard FOD approach are quite robust to sampling variations.

[[[Figure_6, see page 31]]]

4.3. Comparison between poverty indices and FOD approaches

When comparing among the different cardinal and ordinal poverty indices presented in section 4.1 we could rely on widely used statistical association tools: scatterplots, association coefficients and the like (see Figures 4 and 5). However, the fact that the FOD approach generates a partial (i.e. incomplete) preorder among the set of countries in terms of poverty does not allow for a direct comparison with the complete ordering generated by poverty indices. As is clear, comparisons concerning the ranking of couples of countries cannot be straightforwardly established if one criterion is always able to rank them (i.e.: the poverty indices) while the other is not (the FOD approach). In this paper we suggest two possible alternatives to explore the relationship between both approaches. In the first one we restrict our attention to the couples of countries that *can* be ranked according to the FOD criterion, and within this universe we compare the results with the ones that are obtained from the poverty indices shown in section 4.1. The second alternative is to derive a complete preorder from $\leq_{FOD}$ in a 'reasonable' manner so that it is fully comparable with respect to the rankings generated by the poverty indices. We will now examine these two proposals separately in sections 4.3.1 and 4.3.2.

4.3.1. Focusing on comparable and non-comparable pairs

As mentioned before, there are 38·37/2=703 possible pair-wise comparisons among couples of countries. Among these, we see from Table 3 that $\leq_{FOD}$ is able to rank 350 couples, that is: 49.78% of the total. In this subsection we will restrict our attention to this set – which, following [4], is denoted as C_{FOD} – and its complement N_{FOD}, which consists of 703 – 350 = 353 pairs.

To start with, we will check whether the preorder $\leq_{FOD}$ is consistent with the complete orderings generated by the poverty indices considered in this paper: $\leq_{M0}$, $\leq_{M1}$, $\leq_{M2}$, $\leq_{BC21}$ and $\leq_{BC12}$ (see definition [5]). In other words, if there is a couple of countries A, B such that $A \leq_{FOD} B$, we want to verify whether $P(A) \geq P(B)$, where P is the corresponding poverty index. When this happens, we say that the couple of countries (A, B) is a *concordant pair*. Analogously, if one has that $A \leq_{\text{FOD}} B$ but it turns out that $P(A) \leq P(B)$ then the couple (A, B) is called a *discordant pair*. The notions of concordant and discordant pairs are borrowed from the definition of Kendall's tau[13]. For the different poverty measures explored in this paper (M_0, M_1, M_2, $BC_{2,1}$ and $BC_{1,2}$), the shares of concordant pairs within the universe of 350 comparable couples are 100%, 96.86%, 95.43%, 96% and 96.57% respectively. In other words, when a country A is determined to be at least as poor as country B according to the FOD criterion, it is very likely that the poverty level of A – as measured with the poverty indices considered in this paper – will not be lower than the poverty level of B. Therefore, the partial order generated by the FOD criterion is faithfully represented by the different poverty indices explored in this paper. Indeed, when using the ordinal poverty index M_0 the agreement between $\leq_{M0}$ and $\leq_{FOD}$ is complete: both criteria rank *all* couples in C_{FOD} in the same direction.

The previous results explored the pair-wise agreements between $\leq_{FOD}$ and the five poverty preorders $\leq_P$ considered in this paper *separately*. Likewise, we can investigate the *simultaneous* agreement between our five poverty measures on the set of couples belonging to C_{FOD}. It turns out that in 95.43% of the couples of countries belonging to that set, the ranking provided by all five poverty indices is entirely consistent. If we now investigate the level of agreement among the five poverty indices within the set of couples that are not classifiable according to $\leq_{FOD}$ (N_{FOD}), we obtain a much lower percentage of 72.8%. As can be expected, the agreement between the five poverty indices is more common among those couples of countries that first order dominate one way or another than among those couples where the first order dominance criterion does not support a pair-wise ranking. As in section 4.1, it is revealing to examine what happens to the degree of simultaneous agreement among the five poverty measures

[13] Let $(x_1,y_1),\ldots,(x_n,y_n)$ be a set of observations of the joint random variables X and Y. Assuming there are no ties, Kendall's tau is defined as $\tau:=(C–D)/(n(n–1)/2)$, where C (resp. D) is the number of concordant (resp. discordant) pairs of observations and $n(n–1)/2$ is the total number of pair combinations. When all couples of observations are consistently ranked by X and Y, $\tau=1$ and when all couples of observations are inconsistently ranked, $\tau = –1$.

when one of them is dropped from the list at a time. When we look at the simultaneous level of agreement reached between M_1, M_2, $BC_{2,1}$ and $BC_{1,2}$ (that is, when M_0 is dropped from the list) in the sets C_{FOD} and N_{FOD}, we obtain 98.28% and 90.65% respectively. Dropping M_1, M_2, $BC_{2,1}$ and $BC_{1,2}$, the corresponding levels of agreement in C_{FOD} and N_{FOD} are (95.43%, 72.8%), (95.43%, 74.22%), (95.43%, 72.8%), and (95.42%, 73.09%) respectively. In other words: when a single cardinal index is removed from the list, the agreement between the remaining four remains virtually unchanged – either in C_{FOD} or in N_{FOD}. However, dropping the ordinal index M_0 from the list results in an extremely high agreement between the cardinal indices – even when restricting our attention to the set of couples of countries where the first order dominance criterion is inconclusive.

As we have seen, the results of the different poverty indices tend to be more consistent when we restrict our attention to the set of couples of countries where the FOD approach is conclusive (C_{FOD}). In this regard, one might suspect that the differences in poverty levels for the couples of countries included in this group are significantly bigger than the differences in poverty levels for the couples of countries included in N_{FOD}. In technical terms, one might expect that the absolute difference $|P(A) – P(B)|$ should be smaller when $(A,B) \in N_{FOD}$ than when $(A,B) \in C_{FOD}$. In order to investigate this issue, we have generated two density functions of the values of $|P(A) – P(B)|$: one for the couples $(A,B) \in N_{FOD}$ (which is denoted as $f_{N,P}$) and another one for the couples $(A,B) \in C_{FOD}$ (which is denoted as $f_{C,P}$). Figures 7 and 8 show the density functions $f_{N,P}$ and $f_{C,P}$ for the poverty indices M_0 and BC_{21} respectively[14]. As can be seen, both figures indicate that the couples belonging to N_{FOD} tend to have lower absolute difference in poverty levels $|P(A) – P(B)|$ than those couples belonging to C_{FOD}. The medians of $f_{N,P}$ and $f_{C,P}$ in Figure 7 are 0.097 and 0.269 respectively while in Figure 8, the respective medians are 0.073 and 0.157. As expected, the higher the difference in poverty levels between countries A and B, the higher the probability that these countries can be ordered by $\leq_{FOD}$. However, there is a non-negligible amount of couples of countries for which a large difference in poverty levels is not enough to guarantee a concluding poverty assessment in terms of $\leq_{FOD}$, and similarly quite a number of country couples where even small poverty differences results in FOD. Interestingly, the association between having larger differences in poverty levels and being ranked in terms of the FOD approach is more neatly defined in the case of M_0 than in the case of $BC_{2,1}$. Again, this might respond to the fact that the ordinal poverty index M_0 mimics more closely the behavior of the $\leq_{FOD}$ preorder.

[[[Figures_7_and_8, see page 32]]]

4.3.2. Extending comparability

So far, we have analyzed the performance of the poverty indices either in the subspace of couples of countries where the FOD criterion applies (C_{FOD}) or in the one where it

[14] The results for the other cardinal poverty indices are not substantially different, so they have not been shown here to save space.

does not (N_{FOD}). Besides restricting our attention to these two subspaces separately, an alternative way of comparing poverty indices and the FOD approach is to generate a 'reasonable' *complete* preorder from the latter so that the two perspectives can be fully comparable across the entire set of couples of countries ($X \times X$). A straightforward way of generating such a complete preorder from the information provided by $\leq_{FOD}$ is to simply count, for each country $x \in X$, the number of countries $y \in X$ such that $y \leq_{FOD} x$ (that is: to count the number of countries that each country dominates). In technical terms, this is the cardinality of the down-set of x: $|x^{\downarrow}|$. This way, each country gets assigned a cardinal score with which its performance can be assessed: the higher the value of $|x^{\downarrow}|$, the higher the relative welfare in country x. The empirical distribution of those values for the case at hand is shown in the last column of Table 3 (a marginal distribution).

At this point, one might argue that another equally reasonable way of assessing the performance of countries in terms of $\leq_{FOD}$ is to count the number of countries that dominate a given country (that is: consider the values of $|x^{\uparrow}|$ for the different $x \in X$). In this case, the higher the value of $|x^{\uparrow}|$ the lower the relative welfare in country x. The empirical values of that distribution for the case at hand are shown in the last row of Table 3 (a marginal distribution). If one wants the two criteria to classify countries in the same direction (e.g. higher values denoting higher poverty / lower welfare) one can simply rewrite the first criterion and classify countries depending on the values of $|X| - |x^{\downarrow}|$. Interestingly, it turns out that even if the $|X| - |x^{\downarrow}|$ and $|x^{\uparrow}|$ criteria run in the same direction, they do not rank all countries in a consistent way. As shown in Figure 9, the association between both criteria is not 'perfect': the relative position of certain countries vis-à-vis others depends on which criterion is being chosen. Since there are no compelling reasons to choose one of them in detriment of the other, we pragmatically summarize the two criteria by means of a simple average. Therefore, the criteria we derived from FOD in order to assess countries lack of welfare is given by the values of

$$P_{FOD}(x) = \frac{(|X| - 1) - |x^{\downarrow}| + |x^{\uparrow}|}{2 \cdot (|X| - 1)} \qquad [6]$$

where x is any country in X. The subtractions and multiplication of constants in [6] are solely intended to normalize the values of $P_{FOD}(x)$ between 0 and 1: $P_{FOD}(x)$ takes a value of 0 when x first order dominates all countries in X and is dominated by none and it takes a value of 1 when x is first order dominated by all countries and dominates none. As can be seen, P_{FOD} is a relative measure of welfare that essentially compares countries' relative positions abstracting from absolute levels of welfare. The way in which it is constructed P_{FOD} can be thought as an adaptation of the Borda count rule for the incomplete rankings generated by $\leq_{FOD}$.

[[[Figure_9, see page 33]]]

As is clear, the new index P_{FOD} generates a complete preorder on the set of countries X, so it is fully comparable with the other poverty indices introduced in section 2.1. In Figure 10, we show a matrix with the scatterplots comparing the rankings associated to the poverty measures M_0, M_2 and P_{FOD} (we have not introduced the measures M_1, $BC_{2,1}$ and $BC_{1,2}$ because their behavior resembles very much that of M_2 – see Figure 4). The scatterplots of Figure 10 suggest that the indices that are farther apart from each other (i.e. showing a larger dispersion and inconsistency) are P_{FOD} and the cardinal measure M_2. At the other extreme, we have P_{FOD} and the ordinal measure M_0, which are not related in a strictly linear fashion but do not show large discrepancies either.

This visual impression is confirmed by looking at the association coefficients shown in Table 4. In that table we present the classical correlation coefficient, the rank correlation coefficient and Kendall's tau coefficient of statistical association (see Footnote #13) between all poverty measures analyzed in this paper. As can be deduced from that Table, the measures M_1, M_2, $BC_{2,1}$ and $BC_{1,2}$ form a quite homogeneous group that can be denoted as 'cardinal measures cluster' (within that group, correlation coefficients are always above 0.96). At the other extreme, the index P_{FOD} forms the so-called 'FOD cluster'. In the middle of the previous two clusters we find the ordinal measure M_0, which is a kind of mixture between the two: it has the ordinal variables used in the FOD approach but it is constructed with the aggregation methodology that characterizes the cardinal measures. When comparing measures from the cardinal and the FOD clusters the association coefficients are not extremely high (for instance: the correlation coefficient between M_2 and P_{FOD} is 0.79) because for some countries there are important variations in the rankings that ensue from the different measures (see Figure 10). To illustrate, Bangladesh, Nepal, Rwanda, Cambodia and Nigeria change 19, 15, 11, 10 and 10 positions respectively (out of 38) when moving from the M_2 ranking to the P_{FOD} ranking. Interestingly, the association between P_{FOD} and the ordinal measure M_0 used by UNDP is quite high, with a correlation coefficient of 0.95, a result with important implications that will be discussed in the following section.

[[[Figure_10, see page 33]]]

[[[Table_4, see page 28]]]

5. Concluding Remarks

In this paper we have contrasted different approaches to the measurement of multidimensional poverty from an international perspective. Using data from 38 Demographic and Health Surveys around the developing world, we have compared the performance of two broad approaches: the use of multidimensional poverty indices (the most popular approach implemented so far) and the use of first order stochastic dominance techniques (FOD) – a relatively new and seemingly cost-effective approach that does not rely on the host of debatable assumptions upon which poverty indices are typically based. This exercise is particularly relevant in a moment where there are intense debates in the international research community on the most appropriate way of conceptualizing and measuring poverty and that are taking place against a backdrop where the 2015 MDGs deadline is rapidly approaching. This paper attempts to highlight the advantages and disadvantages of currently existing methods to inform and illuminate somehow these debates. To the best of our knowledge, there is currently no assessment of the degree of consistency that might exist between both approaches – an issue we have attempted to address in this work.

The comparison between multidimensional poverty indices and the FOD approach is not straightforward because the latter might not be conclusive when making comparisons among certain couples of countries (indeed, this is the case in 50% of all possible country pairs considered in this paper). In front of this problem, we have opted for a two-pronged strategy to maximize comparability among methodologies. The first strategy consists in analyzing the behavior of the different poverty indices when we either restrict our attention to the set of couples of countries that can be ranked according to the FOD criterion or to its complement of country pairs where FOD is inconclusive. In the former case, it turns out that the agreement among the different poverty indices is very high and consistent with the FOD ordering: in more than 95% of the cases, all poverty indices considered here agree with the pair-wise rankings suggested by FOD. In conclusion, the partial order generated by the FOD criterion is faithfully represented by the different poverty indices explored in this paper. When restricting our attention to the pairs of countries that FOD is unable to rank we observe that the rankings generated by the poverty indices considered here agree unanimously only in 73% of the cases. Since lack of FOD among a couple of countries indicates that different normative criteria can lead to opposite welfare conclusions, the abovementioned reduction in agreement between poverty index rankings is to be expected.

In order to facilitate comparability between both approaches, the other strategy followed in this paper is to extend and complete the partial order generated by FOD in a 'reasonable' manner so that it is fully comparable with respect to the rankings generated by the poverty indices. For that purpose, we have created a relative welfare measure (denoted as P_{FOD}) that, for a given country *x*, essentially averages the number of countries that first-order-dominate *x* and the number of countries that *x* does not first-order-dominate. When comparing P_{FOD} with the rest of poverty indices investigated in this paper two distinct findings arise. On the one hand, while P_{FOD} and the *cardinal* poverty indices investigated in this paper roughly go in the same direction and they broadly paint the same overall picture, their differences are non-negligible: the corresponding correlation coefficients can be below 0.8 and for some countries the ranking variations are quite substantial (e.g. moving up to 19 positions out of a maximum of 37). Therefore, even if both groups of measures offer a roughly consistent assessment of international poverty levels, particular care should be taken to avoid using them indiscriminately – particularly when dealing with countries in the middle of the distribution where variations can be potentially large. On the other hand, the rankings generated by P_{FOD} and the *ordinal* poverty index M_0 (a measure that essentially mimics UNDP's Multidimensional Poverty Index – MPI) are very similar: the correlation coefficient equals 0.95, a remarkably good fit. This important finding suggests that the FOD approach might be a reasonable cost-effective alternative to the MPI in the absence of the data sources that might be necessary to compute UNDP's measure. To the extent that the FOD approach is able to uncover the socio-economic gradient that exists between countries, it can be postulated as a viable alternative to the MPI that has the advantage of not having to rely on the somewhat arbitrary and normatively binding assumptions that underpin the construction of UNDP's index.

Future research may explore the extent to which the FOD approach is also able to uncover territorial variations *within* countries to identify the regions where underdevelopment and social disadvantage are more entrenched. In this regard, it would be interesting to determine if the within-country assessments generated by the FOD approach are consistent with the ones generated by more widely used measures like the multidimensional poverty indices discussed in this paper.

Acknowledgements: Financial support from the Spanish Ministry of Economy and Competitiveness "Ramón y Cajal" Research Grant Program and research projects ERC-2009-StG-240978 and ECO2013-46516-C4-1-R is gratefully acknowledged.

References

Alkire, S. and Foster, J. (2011), "Counting and multidimensional poverty measurement", *Journal of Public Economics* 95 (7-8): 476-487.

Alkire, S. and Santos, M. (2010), "Acute Multidimensional Poverty: A New Index for Developing Countries", Human Development Research Paper 2010/11.

Araar, A., Levine, S. and Duclos, JY. (2009), "Body Mass Index, Poverty and Inequality in Namibia", Paper prepared for Central Bureau of Statistics, Namibia (available at http://www.the-eis.com/data/literature/Namibia_nutrition_2009.pdf).

Arndt, C., Distante, R., Hussain, M.A., Østerdal, L.P., Huong, P. and Ibraimo, M., (2012), "Ordinal Welfare Comparisons with Multiple Discrete Indicators: A First Order Dominance Approach and Application to Child Poverty", *World Development* 40(11): 2290-2301.

Atkinson, A. (2003), "Multidimensional deprivation: contrasting social welfare and counting approaches", *Journal of Economic Inequality* 1:51-65.

Bourguignon, F. and Chakravarty, S. (2003), "The measurement of multidimensional poverty", *Journal of Economic Inequality* 1:25-49.

Chakravarty, S., Deutsch, J. and Silber, J. (2008), "On the Watts Multidimensional Poverty Index and its Decomposition", *World Development* 36(6):1067-1077.

Cherchye, L., Ooghe, E., and Puyenbroeck, T. (2008), "Robust human development rankings", *Journal of Economic Inequality* 6:287–321.

Deutsch, J. and Silber, J. (2005), "Measuring multidimensional poverty: An empirical comparison of various approaches", *Review of Income and Wealth* 51(1): 145-174.

Duclos, J.-Y., Sahn, D. and Younger, S. (2006), "Robust multidimensional poverty comparisons", *The Economic Journal* 116: 943-968.

Filmer, D. and Pritchett, L. (2001), "Estimating Wealth Effects Without Expenditure Data---Or Tears: An Application to Educational Enrollments in States of India", *Demography* 38(1): 115-132.

Filmer, D. and Scott, K. (2012), "Assessing asset indices", *Demography* 49: 359-392.

Foster, J., Greer, J. and Thorbecke, E. (1984), "A class of decomposable poverty measures", *Econometrica* 52(3):761-766.

Foster, J., McGillivray, M. and Seth, S. (2013), "Composite indices: Rank robustness, statistical association, and redundancy", *Econometric Reviews* 32(1):35-56.

Grimm, M., Harttgen, K., Klasen, S. and Misselhorn, M. (2008), "A Human Development Index by Income Groups", *World Development* 36 (12): 2527-2546.

Harttgen, K. and Klasen, S. (2011), "A Household-based Human Development Index", *World Development* 40(5): 878-899.

Lehmann, E. (1955), "Ordered families of distributions", *The Annals of Mathematical Statistics* 26(3):399-419.

McLaren, L. (2007), "Socioeconomic Status and Obesity", *Epidemiologic Reviews* 29:29-48.

Permanyer, I. (2011), "Assessing the Robustness of Composite Indices Rankings", *Review of Income and Wealth* 57(2): 306-326.

Permanyer, I. (2012), "Uncertainty and Robustness in Composite Indices Rankings", *Oxford Economic Papers* 64(1):57-79.

Permanyer, I. (2013), "Using Census Data to Explore the Spatial Distribution of Human Development", *World Development* 46:1-13.

Permanyer, I. (2014), "Assessing individuals' deprivation in a multidimensional framework", *Journal of Development Economics* 109:1 -16.

Ravallion, M. (2011), "On Multidimensional Indices of Poverty", *Journal of Economic Inequality* 9:235-248.

Sahn, D. and Stifel, D. (2000), "Poverty comparisons over time and across countries in Africa", *World Development* 28(12): 2123-2155.

Sahn, D. and Stifel, D. (2003), "Exploring Alternative Measures of Welfare in the Absence of Expenditure Data", *Review of Income and Wealth* 49(4): 463-489.

Sahn, D. and Younger, S. (2009), "Measuring intra-household health inequality: Explorations using the Body Mass Index", *Health Economics* 18: S13-S36.

Sen, A. (1976), "Poverty: An ordinal approach to measurement", *Econometrica* 42 (2): 219-231.

Sobal, J. and Stunkard, A. (1989), "Socioeconomic Status and Obesity: A Review of the Literature", *Psychological Bulletin* 105(2):260-275.

Strassen, V. (1965), "The existence of probability measures with given marginals", *The Annals of Mathematical Statistics* 36(2):423-439.

Thorbecke, E. (2007), "Multidimensional Poverty: Conceptual and Measurement Issues", Chapter 1 in "The Many Dimensions of Poverty", edited by Nanak Kakwani and Jacques Silber, Palgrave Macmillan.

Tsui, K. Y. (2002), "Multidimensional poverty indices", *Social Choice and Welfare* 19: 69-93.

Wittenberg, M. (2013), "The weight of success: The Body Mass Index and economic well-being in Southern Africa", *Review of Income and Wealth* 59 (Special Issue): S62-S83.

Tables

Country	M0	M1	M2	BC21	BC12	UNDP's MPI
Albania	0.031 (1)	0.016 (1)	0.015 (3)	0.028 (1)	0.005 (3)	0.005 (2)
Armenia	0.060 (3)	0.038 (6)	0.037 (7)	0.065 (7)	0.012 (7)	0.004 (1)
Azerbaijan	0.062 (4)	0.031 (4)	0.030 (6)	0.054 (5)	0.010 (5)	0.021 (5)
Bangladesh	0.406 (27)	0.086 (14)	0.063 (14)	0.135 (15)	0.026 (14)	0.292 (21)
Benin	0.397 (26)	0.160 (27)	0.137 (28)	0.243 (27)	0.064 (27)	0.412 (31)
Bolivia	0.105 (6)	0.031 (5)	0.027 (5)	0.053 (4)	0.009 (4)	0.089 (9)
Cambodia	0.321 (21)	0.087 (15)	0.065 (15)	0.134 (14)	0.029 (16)	0.251 (19)
Cameroon	0.319 (20)	0.143 (26)	0.117 (26)	0.219 (26)	0.054 (26)	0.287 (20)
Colombia	0.118 (7)	0.056 (9)	0.054 (11)	0.097 (9)	0.018 (10)	0.022 (6)
Egypt	0.041 (2)	0.017 (2)	0.015 (2)	0.029 (2)	0.005 (2)	0.024 (7)
Ethiopia	0.644 (37)	0.363 (37)	0.314 (37)	0.483 (37)	0.191 (37)	0.562 (37)
Ghana	0.182 (10)	0.063 (11)	0.052 (10)	0.100 (10)	0.022 (13)	0.144 (11)
Guinea	0.458 (31)	0.214 (32)	0.180 (32)	0.327 (33)	0.081 (30)	0.506 (35)
Haiti	0.337 (23)	0.096 (19)	0.069 (17)	0.142 (16)	0.034 (20)	0.299 (22)
Honduras	0.188 (12)	0.024 (3)	0.010 (1)	0.038 (3)	0.004 (1)	0.159 (14)
Jordan	0.076 (5)	0.060 (10)	0.060 (13)	0.104 (11)	0.020 (12)	0.008 (3)
Kenya	0.308 (19)	0.112 (21)	0.084 (21)	0.177 (21)	0.038 (21)	0.229 (18)
Lesotho	0.236 (15)	0.079 (13)	0.054 (12)	0.133 (13)	0.019 (11)	0.156 (13)
Liberia	0.366 (24)	0.170 (28)	0.138 (29)	0.249 (28)	0.070 (29)	0.485 (34)
Madagascar	0.556 (36)	0.209 (30)	0.163 (30)	0.296 (30)	0.089 (33)	0.357 (26)
Malawi	0.433 (29)	0.175 (29)	0.135 (27)	0.262 (29)	0.067 (28)	0.381 (29)
Maldives	0.142 (9)	0.114 (22)	0.112 (25)	0.195 (24)	0.038 (22)	0.018 (4)
Mali	0.523 (35)	0.274 (36)	0.237 (36)	0.391 (36)	0.125 (36)	0.558 (36)
Namibia	0.269 (17)	0.095 (18)	0.075 (19)	0.155 (20)	0.030 (17)	0.187 (17)
Nepal	0.409 (28)	0.098 (20)	0.066 (16)	0.146 (17)	0.031 (19)	0.35 (25)
Nicaragua	0.185 (11)	0.041 (7)	0.025 (4)	0.062 (6)	0.012 (6)	0.128 (10)
Niger	0.664 (38)	0.399 (38)	0.354 (38)	0.525 (38)	0.219 (38)	0.642 (38)
Nigeria	0.265 (16)	0.122 (24)	0.106 (24)	0.181 (22)	0.053 (25)	0.31 (23)
Peru	0.123 (8)	0.054 (8)	0.047 (9)	0.091 (8)	0.017 (9)	0.086 (8)
Rwanda	0.498 (32)	0.253 (35)	0.212 (35)	0.386 (35)	0.097 (34)	0.426 (32)
Sao Tome & Prin.	0.280 (18)	0.091 (16)	0.073 (18)	0.149 (18)	0.028 (15)	0.154 (12)
Senegal	0.447 (30)	0.210 (31)	0.185 (33)	0.315 (31)	0.089 (32)	0.384 (30)
Sierra Leone	0.500 (33)	0.244 (34)	0.202 (34)	0.338 (34)	0.116 (35)	0.439 (33)
Swaziland	0.208 (14)	0.092 (17)	0.082 (20)	0.151 (19)	0.031 (18)	0.184 (16)
Timor-Leste	0.393 (25)	0.130 (25)	0.099 (23)	0.202 (25)	0.044 (24)	0.36 (27)
Uganda	0.515 (34)	0.216 (33)	0.175 (31)	0.321 (32)	0.087 (31)	0.367 (28)
Zambia	0.327 (22)	0.119 (23)	0.090 (22)	0.185 (23)	0.040 (23)	0.328 (24)
Zimbabwe	0.201 (13)	0.064 (12)	0.046 (8)	0.107 (12)	0.016 (8)	0.18 (15)

Table 1. Ordinal and cardinal multidimensional poverty measures for 38 countries (the corresponding rankings are indicated in parentheses). Source: Authors' calculations using DHS data.

Table 2. *Population distribution by welfare indicator combination. %*

	0/1 indicators by position. 1st: Education. 2nd: Wealth. 3rd: Health.								Total
	000	001	010	011	100	101	110	111	
	Worst	Health	Wealth	We, He	Educati.	Ed, He	Ed, We	Best	
Albania	0.00	0.00	0.18	0.45	0.01	0.09	8.45	90.82	100
Armenia	0.03	0.00	0.04	0.10	0.19	0.82	16.72	82.09	100
Azerbaijan	0.00	0.01	0.05	0.25	0.60	2.95	14.15	81.99	100
Bangladesh	12.14	14.83	4.65	8.46	7.10	10.24	13.73	28.85	100
Benin	7.37	23.68	4.90	20.89	2.70	6.10	7.52	26.84	100
Bolivia	0.22	4.10	0.46	7.68	0.36	5.50	8.13	73.55	100
Cameroon	4.38	17.05	2.58	5.59	4.02	20.80	9.12	36.45	100
Colombia	0.16	1.04	1.22	7.90	0.22	1.02	21.27	67.18	100
Egypt	0.02	0.30	0.22	9.64	0.01	0.26	1.36	88.18	100
Ethiopia	22.42	47.43	0.68	0.84	7.28	11.27	3.23	6.85	100
Ghana	2.17	6.80	1.50	8.40	1.98	8.32	10.82	60.00	100
Guinea	8.27	28.56	7.63	24.53	1.49	4.74	8.12	16.67	100
Honduras	0.50	12.89	0.22	5.70	0.95	17.79	3.48	58.48	100
Haiti	6.84	20.73	2.41	10.90	2.96	7.90	9.70	38.56	100
Jordan	0.00	0.00	0.16	0.88	0.00	0.00	21.66	77.29	100
Kenya	2.73	6.05	0.13	0.73	11.48	42.74	5.55	30.59	100
Cambodia	5.91	12.70	5.30	13.17	3.21	7.05	15.98	36.69	100
Liberia	4.14	27.09	0.61	6.70	3.94	23.82	3.84	29.85	100
Lesotho	0.95	7.97	0.07	0.66	3.87	41.77	1.92	42.78	100
Madagascar	20.30	34.47	1.24	3.72	6.15	11.86	6.68	15.59	100
Mali	11.19	40.97	4.52	17.78	2.10	5.89	4.54	13.00	100
Maldives	0.03	0.00	1.17	2.77	0.10	0.00	37.48	58.45	100
Malawi	6.78	25.58	0.83	3.21	7.93	33.32	4.60	17.75	100
Nicaragua	0.68	16.14	0.14	8.71	0.31	8.10	3.47	62.45	100
Nigeria	5.47	13.11	1.30	4.84	3.60	13.45	8.94	49.28	100
Niger	22.74	56.76	1.26	4.28	1.86	5.02	2.17	5.91	100
Namibia	2.34	5.70	0.57	2.47	10.10	22.32	16.24	40.27	100
Nepal	10.00	19.93	3.17	7.74	8.08	14.51	8.21	28.36	100
Peru	0.44	3.26	0.22	1.87	1.42	11.83	12.10	68.87	100
Rwanda	5.15	44.57	0.08	1.27	3.98	34.57	0.97	9.42	100
Sierra Leone	12.43	35.31	1.37	3.99	6.03	17.92	5.46	17.48	100
Senegal	10.73	16.44	13.64	18.83	1.34	2.48	17.84	18.69	100
Sao Tome & P.	2.96	15.09	1.68	10.50	2.86	12.31	13.18	41.43	100
Swaziland	1.33	4.41	0.99	5.74	5.31	11.62	19.81	50.79	100
Timor-Leste	6.56	11.89	1.12	1.73	15.17	26.68	13.63	23.23	100
Uganda	11.16	18.77	0.29	1.04	22.39	30.98	6.17	9.20	100
Zambia	3.39	13.74	0.30	1.32	8.23	34.89	7.33	30.79	100
Zimbabwe	0.55	2.50	0.05	0.72	5.91	34.81	6.36	49.09	100
Unw. average	5.59	16.05	1.76	6.21	4.35	14.36	10.00	41.68	100

Source: Own calculations based on DHS 2001-2010.

Table 3. *Country FOD comparisons. DHS surveys, 2001-2010. (A '1' in a cell indicates that the row country dominates the column country)*

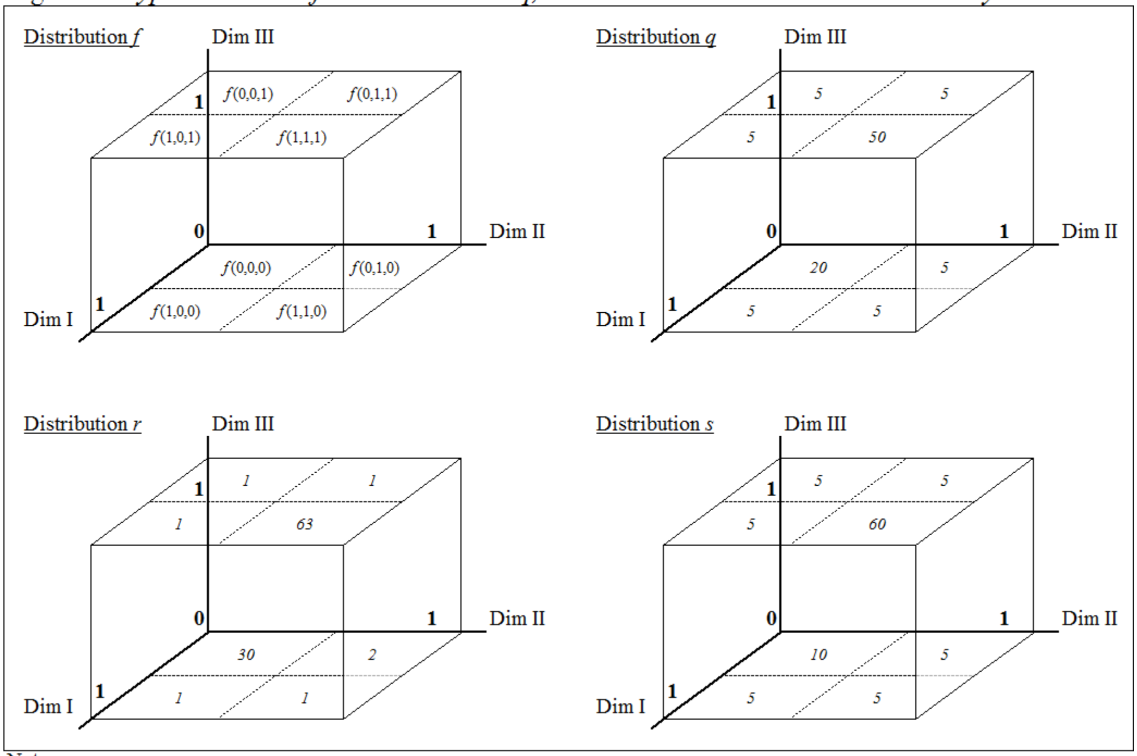
		BD	BJ	BO	CM	CO	ET	GH	GN	HN	HT	KE	KH	LB	MD	ML	MV	MW	NC	NG	NI	NM	NP	PE	RW	SL	SN	ST	SZ	TL	UG	ZM	ZW	All
AL	Albania	1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	30
AM	Armenia	1	1		1	1	1		1		1	1	1		1	1		1		1	1	1	1			1	1	1	1	1	1	1		23
AZ	Azerbaijan	1	1		1		1	1	1		1	1	1		1	1		1		1	1	1	1			1	1	1	1	1	1	1		23
BJ	Benin						1								1						1													3
BO	Bolivia	1	1		1		1	1	1		1		1	1	1	1		1		1	1		1		1	1	1	1	1	1	1	1		23
CM	Cameroon						1								1	1		1			1		1			1					1			8
CO	Colombia	1					1		1				1		1						1	1	1			1	1		1	1	1			13
EG	Egypt	1	1	1	1	1	1	1	1	1	1		1	1	1	1		1	1	1	1	1	1		1	1	1	1	1	1	1	1		28
GH	Ghana	1	1		1		1		1		1		1		1	1		1		1	1		1			1	1	1		1	1			18
GN	Guinea						1														1													2
HN	Honduras	1	1		1		1		1		1			1	1	1		1		1	1		1		1	1		1		1	1			18
HT	Haiti		1				1								1	1					1					1								6
JO	Jordan	1	1			1	1		1		1		1		1	1	1				1	1	1			1	1		1	1	1			18
KE	Kenya						1								1			1			1					1					1			6
KH	Cambodia	1					1								1												1							4
LB	Liberia						1								1	1					1					1								5
LS	Lesotho						1							1	1	1		1			1				1	1				1	1	1		11
ML	Mali																				1													1
MV	Maldives																										1				1			2
MW	Malawi						1														1													2
NC	Nicaragua	1	1		1		1		1		1		1	1	1	1		1			1		1		1	1	1	1			1			18
NG	Nigeria	1	1				1		1		1				1	1		1			1		1			1					1			12
NM	Namibia	1					1								1								1							1	1			6
NP	Nepal						1								1																			2
PE	Peru	1	1		1		1	1	1		1	1	1		1	1		1		1	1	1	1			1	1	1	1	1	1	1		23
RW	Rwanda						1																											1
SL	Sierra Leone						1								1						1													3
ST	Sao Tome & P.	1	1				1		1		1				1	1					1		1			1					1			11
SZ	Swaziland	1					1						1		1						1		1				1			1	1			9
TL	Timor-Leste																														1			1
ZM	Zambia						1								1			1			1					1				1	1			7
ZW	Zimbabwe	1			1		1					1			1	1		1			1		1			1				1	1	1		13
All		17	13	2	10	4	29	5	13	1	12	5	11	6	26	17	2	15	1	8	26	7	17	1	6	20	13	9	8	15	22	8	1	350

	M0	M1	M2	BC21	BC12	FOD
M0	1 1 1					
M1	0.9098 0.9144 0.7724	1 1 1				
M2	0.8687 0.8689 0.6842	0.9949 0.9873 0.9118	1 1 1			
BC21	0.9095 0.8967 0.744	0.9956 0.9947 0.9602	0.99 0.9926 0.9403	1 1 1		
BC12	0.8589 0.8971 0.7383	0.9873 0.9921 0.9374	0.9875 0.9908 0.9403	0.9642 0.9867 0.909	1 1 1	
FOD	0.9471 0.9533 0.825	0.8283 0.8685 0.7112	0.7905 0.8321 0.6458	0.8365 0.8545 0.6885	0.7676 0.86 0.6885	1 1 1

Table 4. Correlation coefficient, Spearman rank correlation coefficient and Kendall's tau coefficient between different poverty measures. Source: Authors' calculations using DHS data.

Figures

Figure 1. *Hypothetical welfare distributions q, r and s in the three dimensional binary case. %*



Notes:
-Dimensions I, II and III are binary welfare indicators with 0 being a bad outcome and 1 being a good outcome.
-Numbers in *italic* are probabilites for the joint distribution of dimensions I-III.
-The 'floor' and the 'roof' represent bad respectively good outcome with repsect to dimension III.
-The best simulataneous outcome is the lower right quadrant on the 'roof', while the worst is the upper left quadrant on the 'floor'.

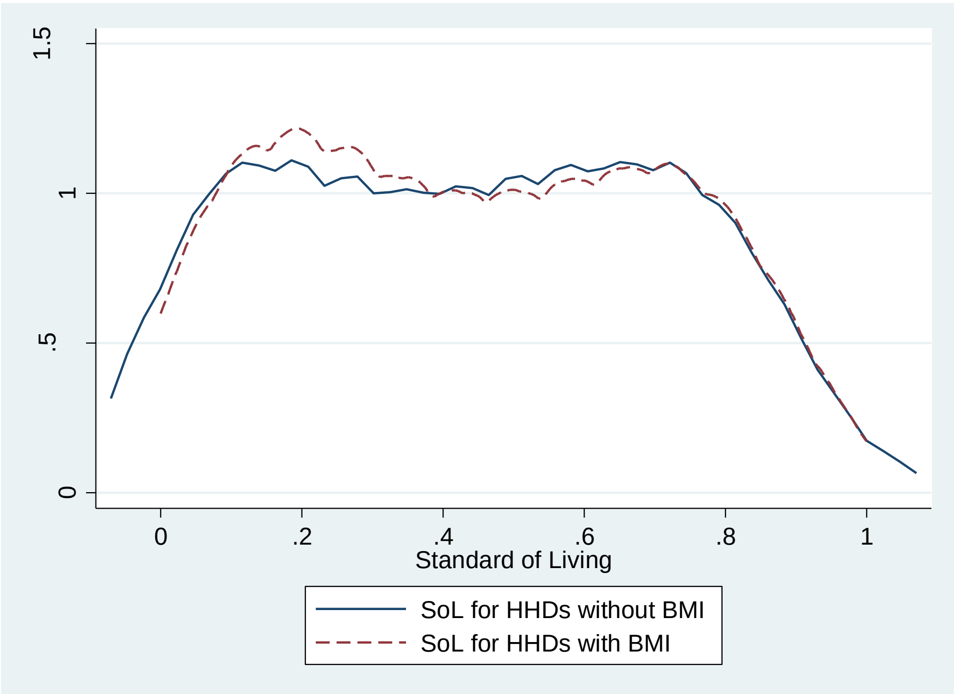


Figure 2. Standard of living (SoL) distributions for households with and without information on the BMI from a pooled dataset consisting of 38 DHS. Source: Authors' calculations using DHS data.

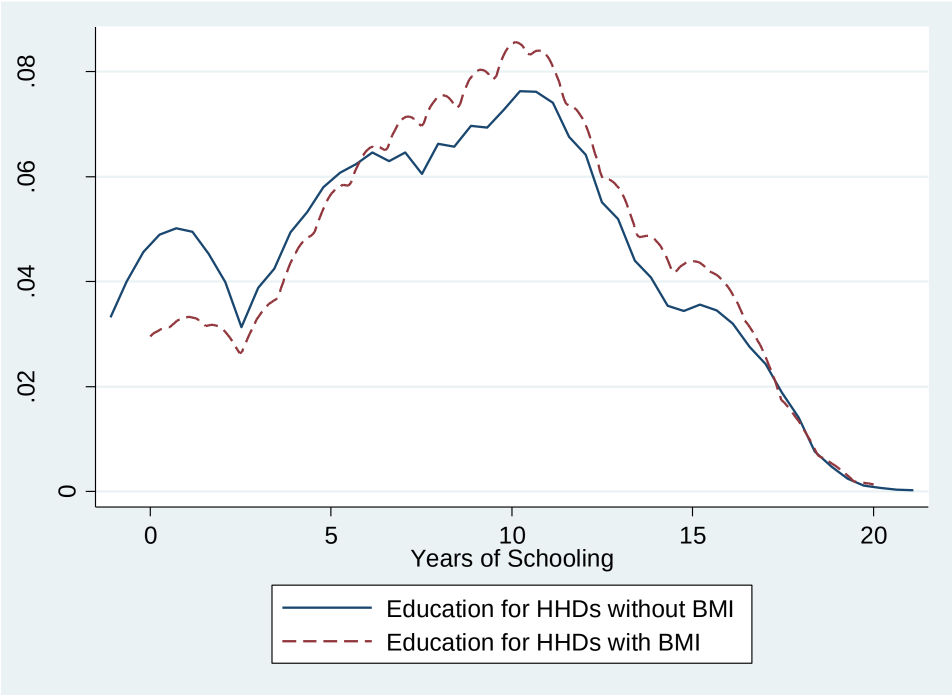


Figure 3. Education distributions for households with and without information on the BMI from a pooled dataset consisting of 38 DHS. Source: Authors' calculations using DHS data.

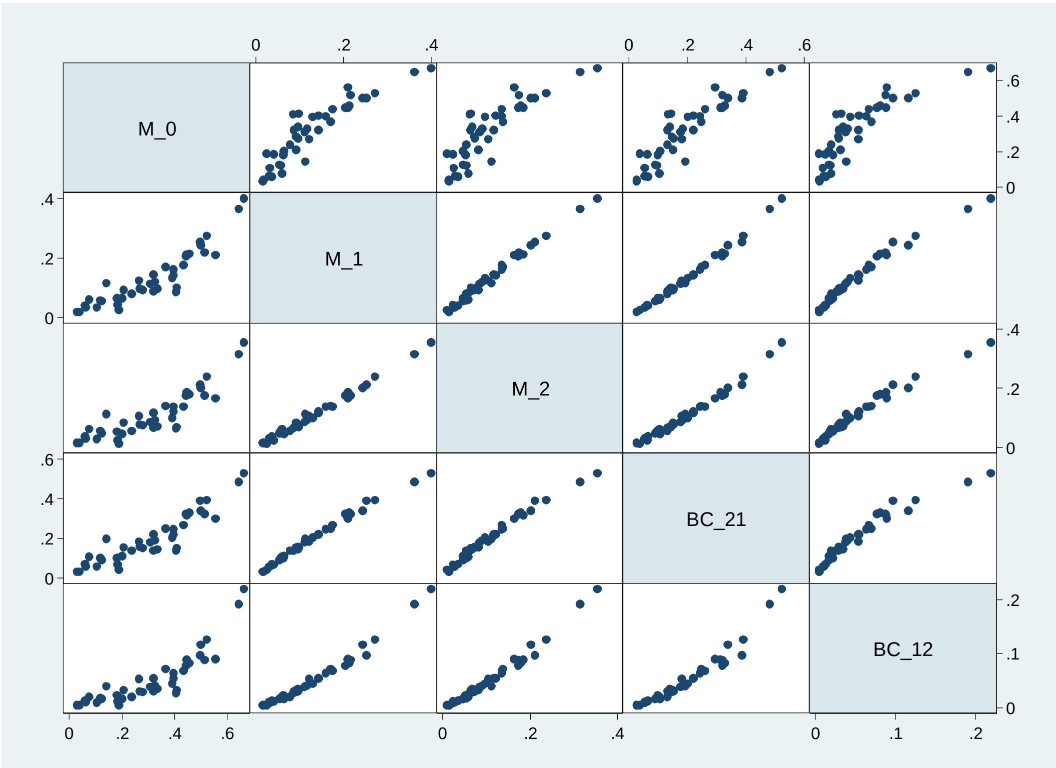


Figure 4. Scatterplot matrix for the cardinal and ordinal measures M_0, M_1, M_2 and BC_{21}, BC_{12}. Authors' calculations using DHS data.

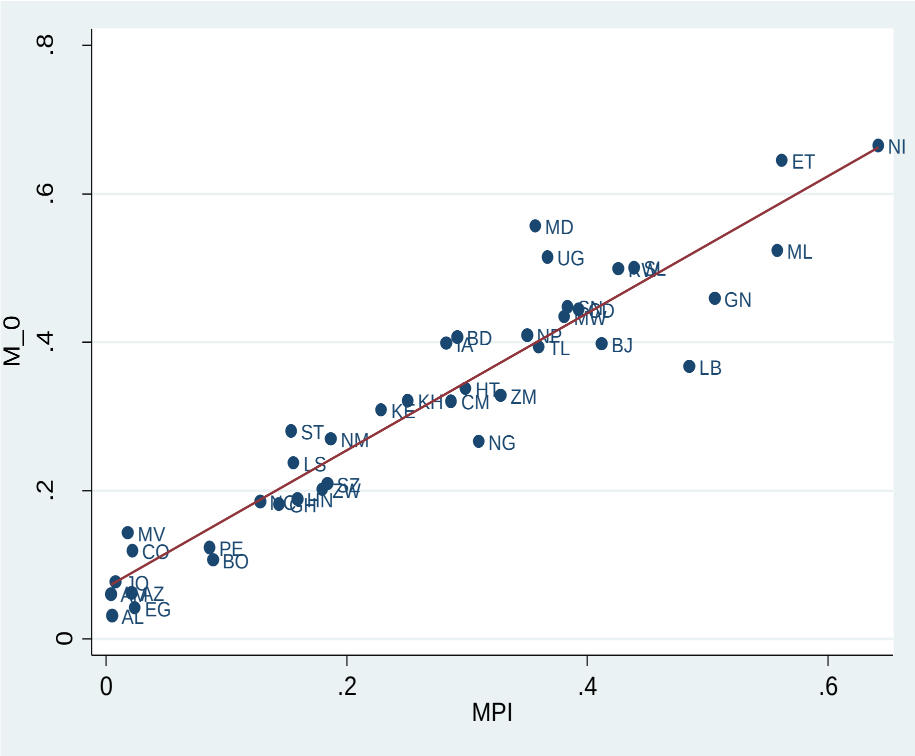


Figure 5. Scatterplot between M_0 and UNDP's MPI. Country labels follow the ISO-3166 coding scheme. Source: Authors' calculations using DHS and UNDP data.

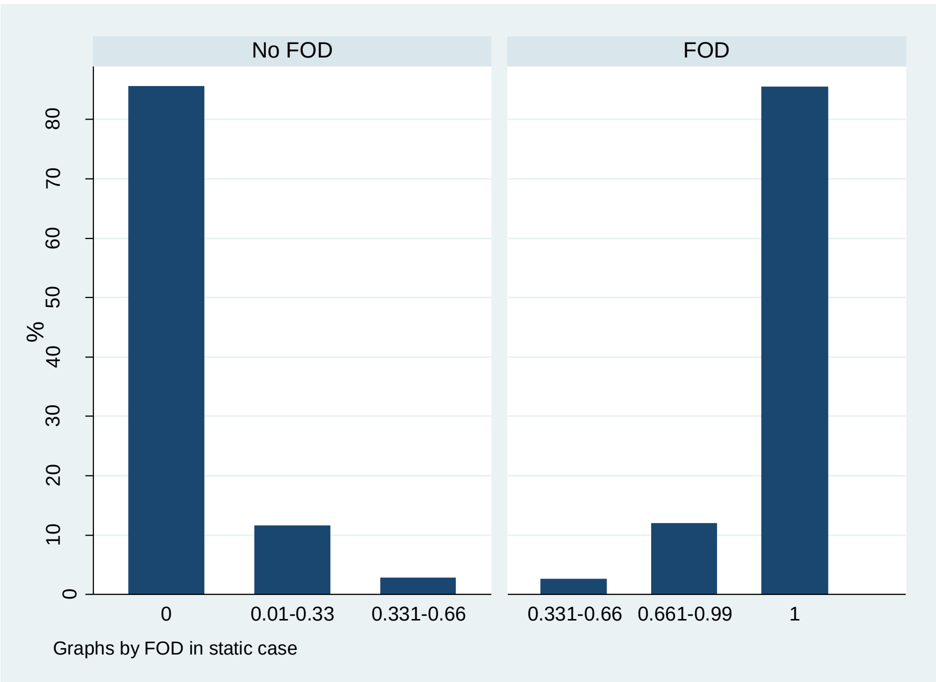


Figure 6. Empirical FOD probabilities from bootstrap, separately by FOD/no FOD status in the static case. Source: Authors' calculations based on DHS data.

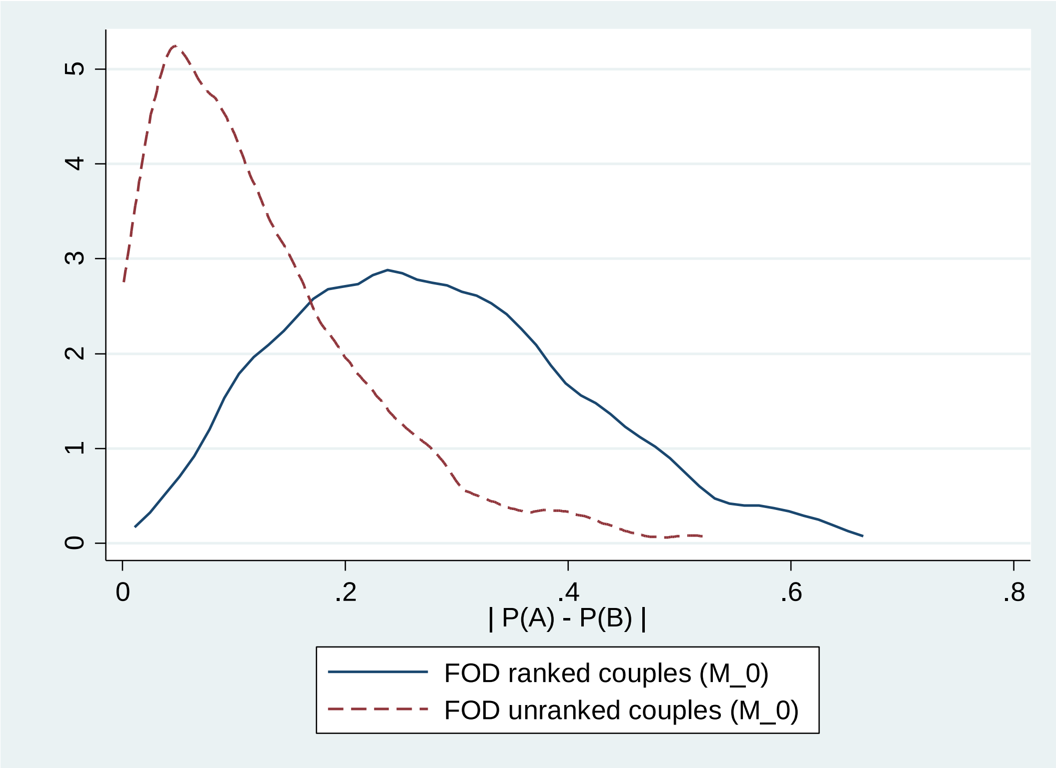


Figure 7. Density functions for the values of $|P(A) - P(B)|$ among FOD ranked and unranked couples when $P=M_0$. Source: Authors' calculations using DHS data.

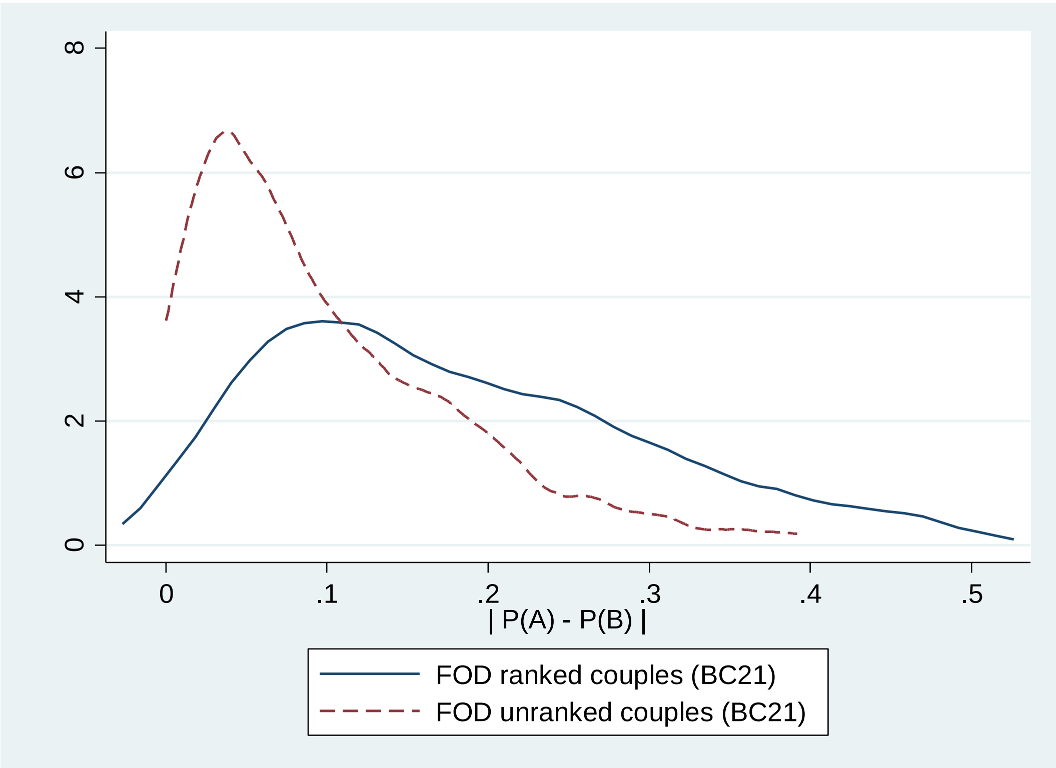


Figure 8. Density functions for the values of $|P(A) - P(B)|$ among FOD ranked and unranked couples when $P=BC_{2,1}$. Source: Authors' calculations using DHS data.

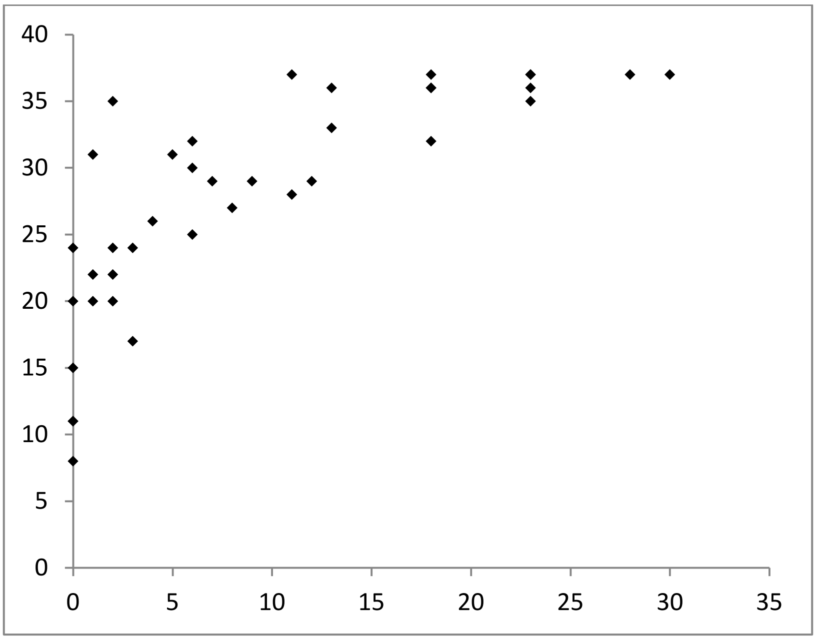


Figure 9. Comparing row and column FOD dominance among the 38 countries in our dataset using data from Table 3. Source: Authors' calculations using DHS data.

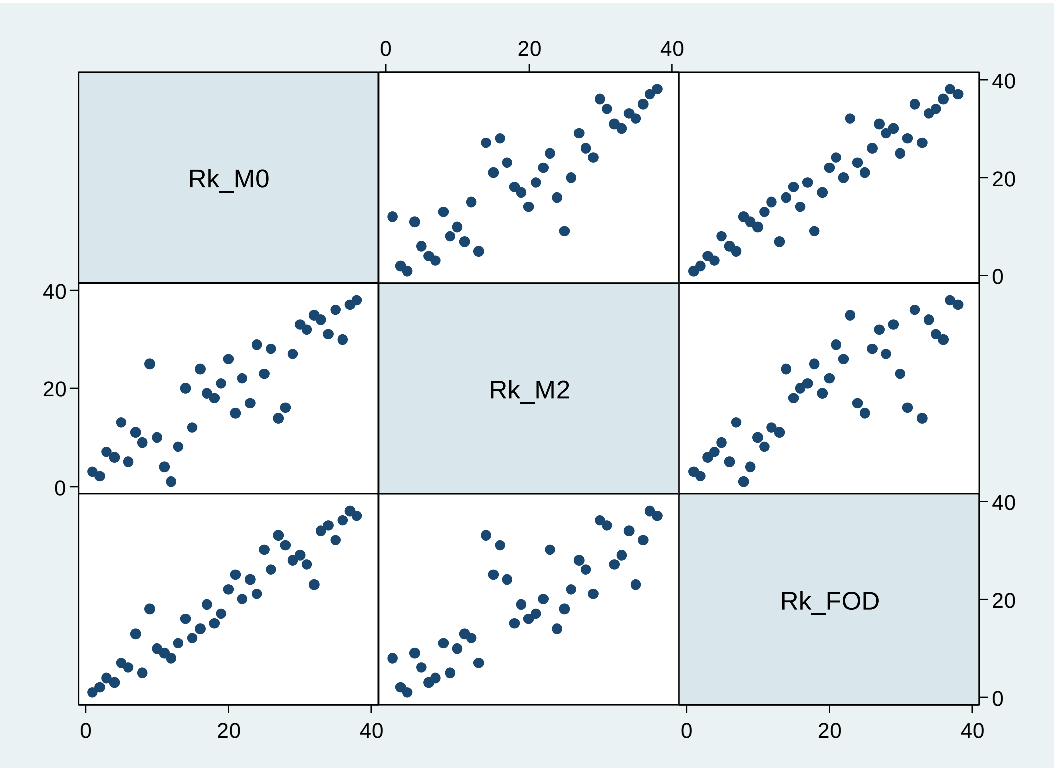


Figure 10. Scatterplot matrix for the rankings associated to M_0, M_2 and P_{FOD}. Source: Authors' calculations using DHS data.